

# Module 2 DC Circuit

# Lesson 6

## Wye (Y) - Delta ( $\Delta$ ) OR Delta ( $\Delta$ )-Wye (Y) Transformations

## Objectives

- A part of a larger circuit that is configured with three terminal network  $Y$  (or  $\Delta$ ) to convert into an equivalent  $\Delta$  (or  $Y$ ) through transformations.
- Application of these transformations will be studied by solving resistive circuits.

### L.6.1 Introduction

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration. A circuit configuration shown below

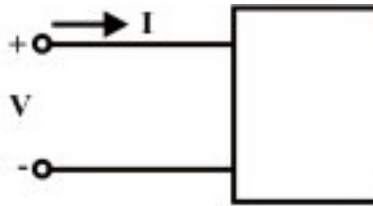


Fig. 6.1(a) One port network

is a general **one-port circuit**. When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a **one-port**. On the other hand, a **two-port** is a circuit having two pairs of terminals. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

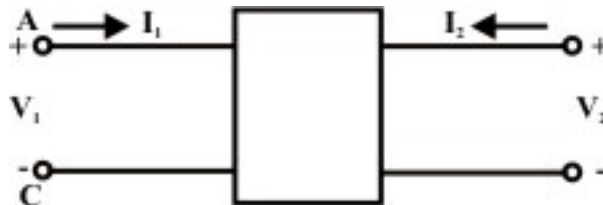
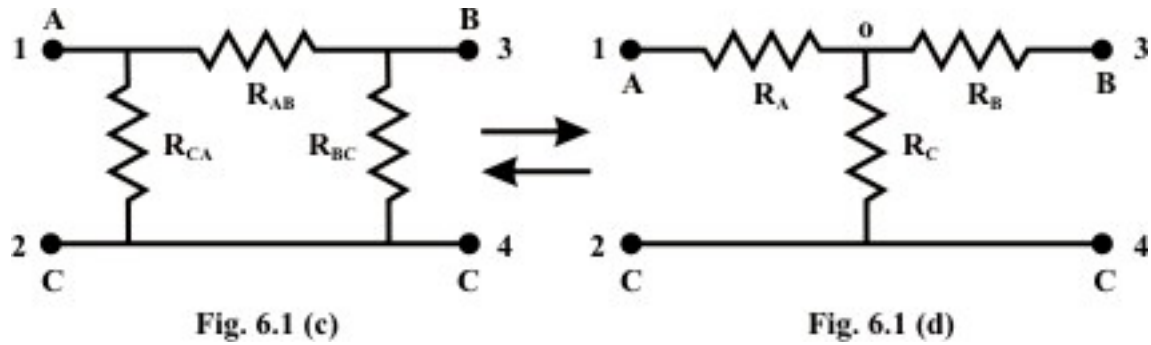


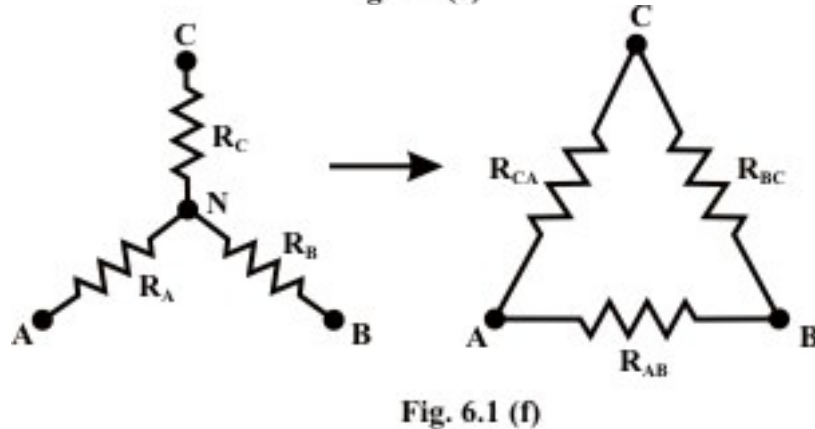
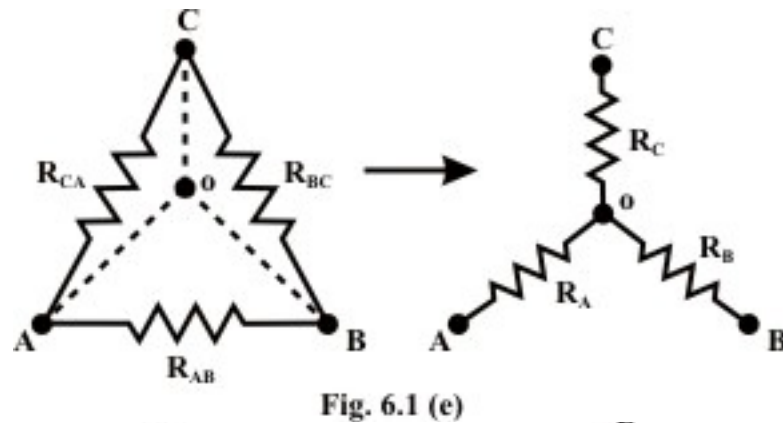
Fig. 6.1(b) Two port network

Fig.6.1.(b) can be described as a four terminal network, for convenience subscript 1 to refer to the variables at the input port (at the left) and the subscript 2 to refer to the variables at the output port (at the right). The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readily possible to consider the ' $\pi$  or  $\Delta$ ' – network also as a three-terminal network in fig.6.1(c). Another frequently encountered circuit configuration that shown in fig.6.1(d) is approximately referred to as a three-terminal  $Y$  connected circuit as well as two-port circuit.



The name derives from the shape or configuration of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter  $\Delta$ .

### L.6.1.1 Delta ( $\Delta$ ) – Wye (Y) conversion



These configurations may often be handled by the use of a  $\Delta - Y$  or  $Y - \Delta$  transformation. One of the most basic three-terminal network equivalent is that of three resistors connected in “Delta( $\Delta$ )” and in “Wye(Y)”. These two circuits identified in fig.L6.1(e) and Fig.L.6.1(f) are sometimes part of a larger circuit and obtained their names from their configurations. These three terminal networks can be redrawn as four-terminal networks as shown in fig.L.6.1(c) and fig.L.6.1(d). We can obtain useful expression for direct

transformation or conversion from  $\Delta$  to  $Y$  or  $Y$  to  $\Delta$  by considering that for equivalence the two networks have the same resistance when looked at the similar pairs of terminals.

## L.6.2 Conversion from Delta ( $\Delta$ ) to Star or Wye ( $Y$ )

Let us consider the network shown in fig.6.1(e) (or fig.6.1(c)  $\rightarrow$ ) and assumed the resistances ( $R_{AB}$ ,  $R_{BC}$ , and  $R_{CA}$ ) in  $\Delta$  network are known. Our problem is to find the values of  $R_A$ ,  $R_B$ , and  $R_C$  in Wye ( $Y$ ) network (see fig.6.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

**Between A & C terminals:**

$$R_A + R_C = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad (6.1)$$

**Between C & B terminals:**

$$R_C + R_B = \frac{R_{BA}(R_{AB} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \quad (6.2)$$

**Between B & A terminals:**

$$R_B + R_A = \frac{R_{AB}(R_{CA} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad (6.3)$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \quad (6.4)$$

Subtracting equations (6.2), (6.1), and (6.3) from (6.4) equations, we can write the express for unknown resistances of Wye ( $Y$ ) network as

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad (6.5)$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad (6.6)$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad (6.7)$$

### L.6.2.1 Conversion from Star or Wye (Y) to Delta (Δ)

To convert a **Wye (Y)** to a **Delta (Δ)**, the relationships  $R_{AB}$ ,  $R_{BC}$ , and  $R_C$  must be obtained in terms of the **Wye (Y)** resistances  $R_A$ ,  $R_B$ , and  $R_C$  (referring to fig.6.1 (f)). Considering the Y connected network, we can write the current expression through  $R_A$  resistor as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad (\text{for } Y \text{ network}) \quad (6.8)$$

Applying KCL at 'N' for Y connected network (assume A, B, C terminals having higher potential than the terminal N) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \Rightarrow V_N \left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right) = \left( \frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)$$

$$\text{or, } \Rightarrow V_N = \frac{\left( \frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)}{\left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \quad (6.9)$$

For Δ-network (see fig.6.1(f)),

Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad (\text{for } \Delta \text{ network}) \quad (6.10)$$

From equations (6.8) and (6.10),

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the  $V_N$  expression in the above equation, we get

$$\frac{\left( V_A - \frac{\left( \frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)}{\left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \Rightarrow \frac{\left( \frac{\left( \frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C} \right)}{\left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

$$\text{or } \frac{\left( \frac{\left( \frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C} \right)}{\left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad (6.11)$$

Equating the coefficients of  $V_{AB}$  and  $V_{AC}$  in both sides of eq.(6.11), we obtained the following relationship.

$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B \left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \quad (6.12)$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C \left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B} \quad (6.13)$$

Similarly,  $I_B$  for both the networks (see fig.61(f)) are given by

$$I_B = \frac{(V_B - V_N)}{R_B} \quad (\text{for } Y \text{ network})$$

$$I_B = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}} \quad (\text{for } \Delta \text{ network})$$

Equating the above two equations and using the value of  $V_N$  (see eq.(6.9), we get the final expression as

$$\frac{\left( \frac{\left( \frac{V_{BC}}{R_C} + \frac{V_{BA}}{R_A} \right)}{\left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_B} = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}}$$

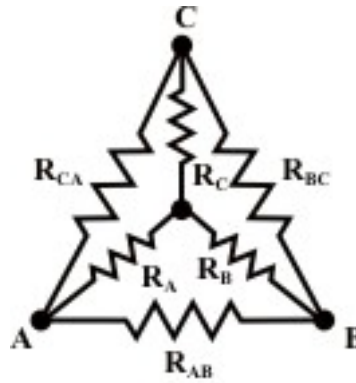
Equating the coefficient of  $V_{BC}$  in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C \left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \quad (6.14)$$

When we need to transform a Delta ( $\Delta$ ) network to an equivalent Wye ( $Y$ ) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye ( $Y$ ) to Delta ( $\Delta$ ) conversion.

### Observations

In order to note the symmetry of the transformation equations, the Wye ( $Y$ ) and Delta ( $\Delta$ ) networks have been superimposed on each other as shown in fig. 6.2.



**Fig. 6.2**

- The equivalent star (Wye) resistance connected to a given terminal is equal to the product of the two Delta ( $\Delta$ ) resistances connected to the same terminal divided by the sum of the Delta ( $\Delta$ ) resistances (see fig. 6.2).
- The equivalent Delta ( $\Delta$ ) resistance between two-terminals is the sum of the two star (Wye) resistances connected to those terminals plus the product of the same two star (Wye) resistances divided by the third star (Wye ( $Y$ )) resistance (see fig.6.2).

### L.6.3 Application of Star ( $Y$ ) to Delta ( $\Delta$ ) or Delta ( $\Delta$ ) to Star ( $Y$ ) Transformation

**Example:** L.6.1 Find the value of the voltage source ( $V_s$ ) that delivers 2 Amps current through the circuit as shown in fig.6.3.



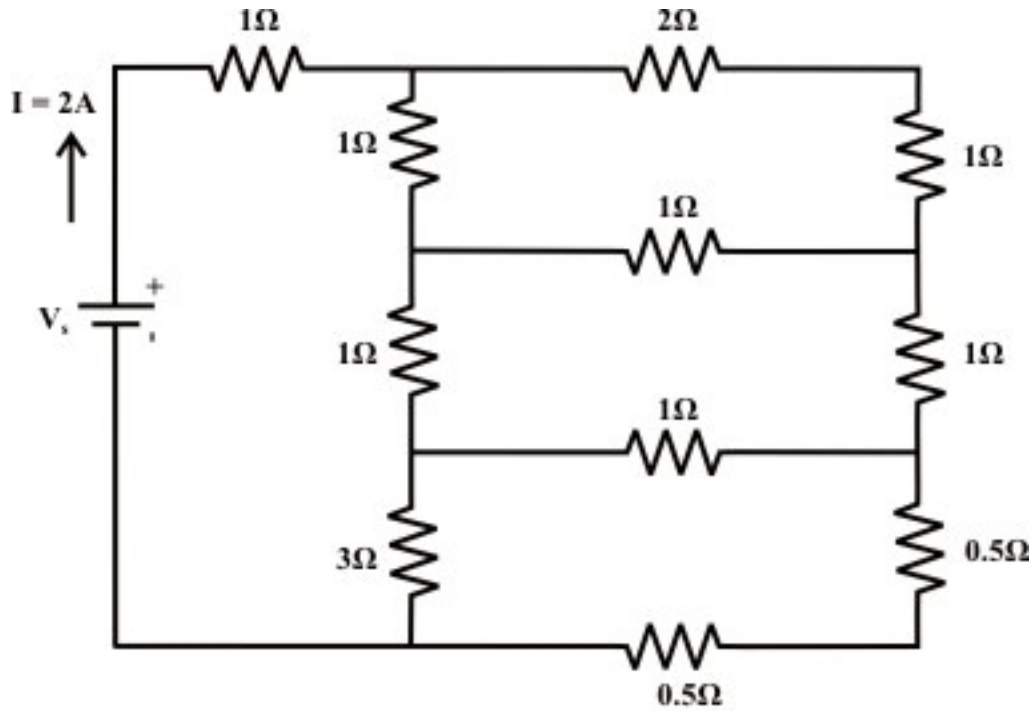
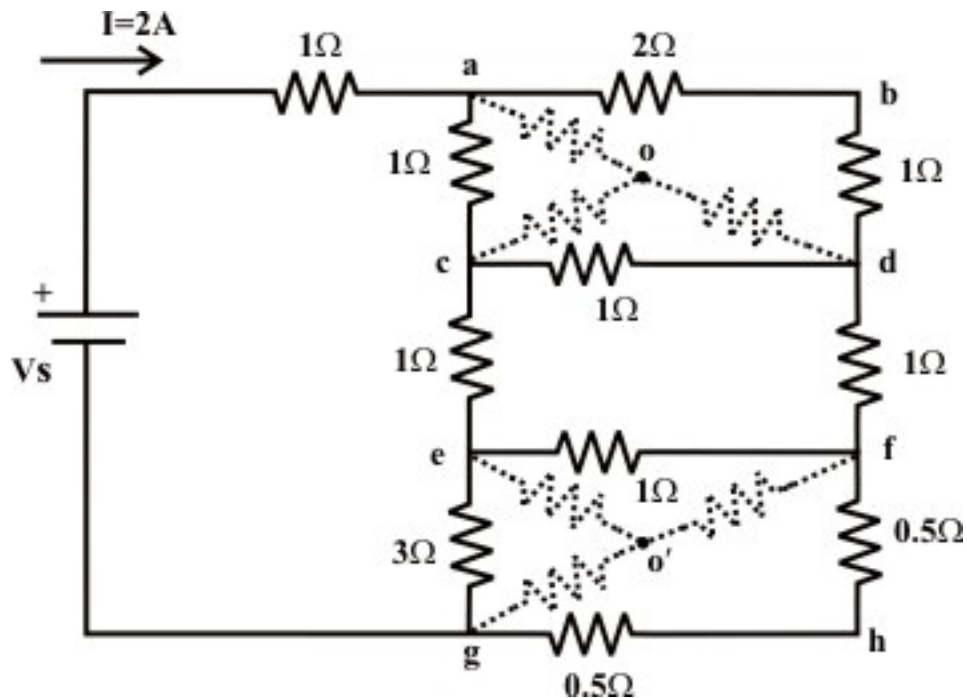


Fig. 6.3

**Solution:**



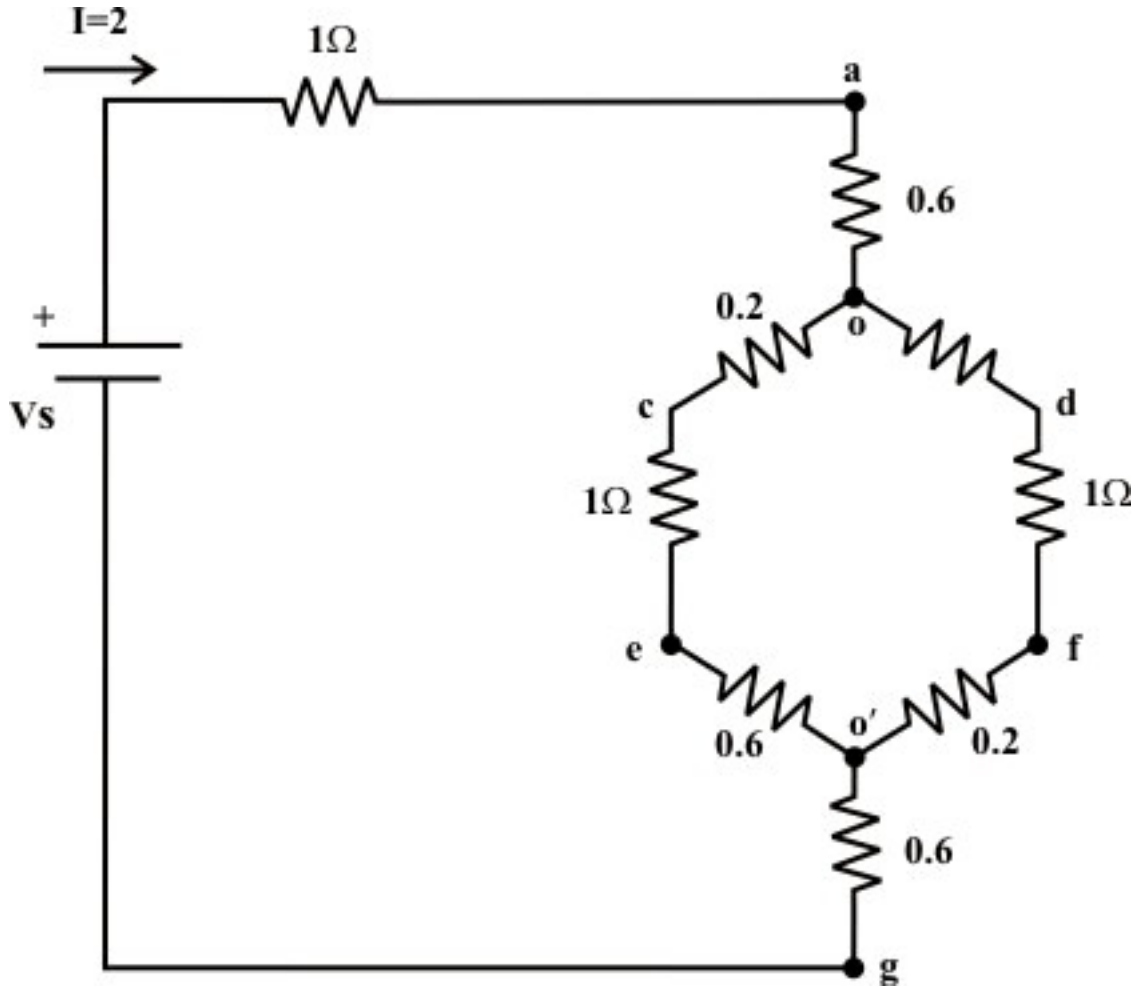
Convert the three terminals  $\Delta$ -network (a-c-d & e-f-g) into an equivalent Y-connected network. Consider the  $\Delta$ -connected network 'a-c-d' and the corresponding equivalent Y-connected resistor values are given as

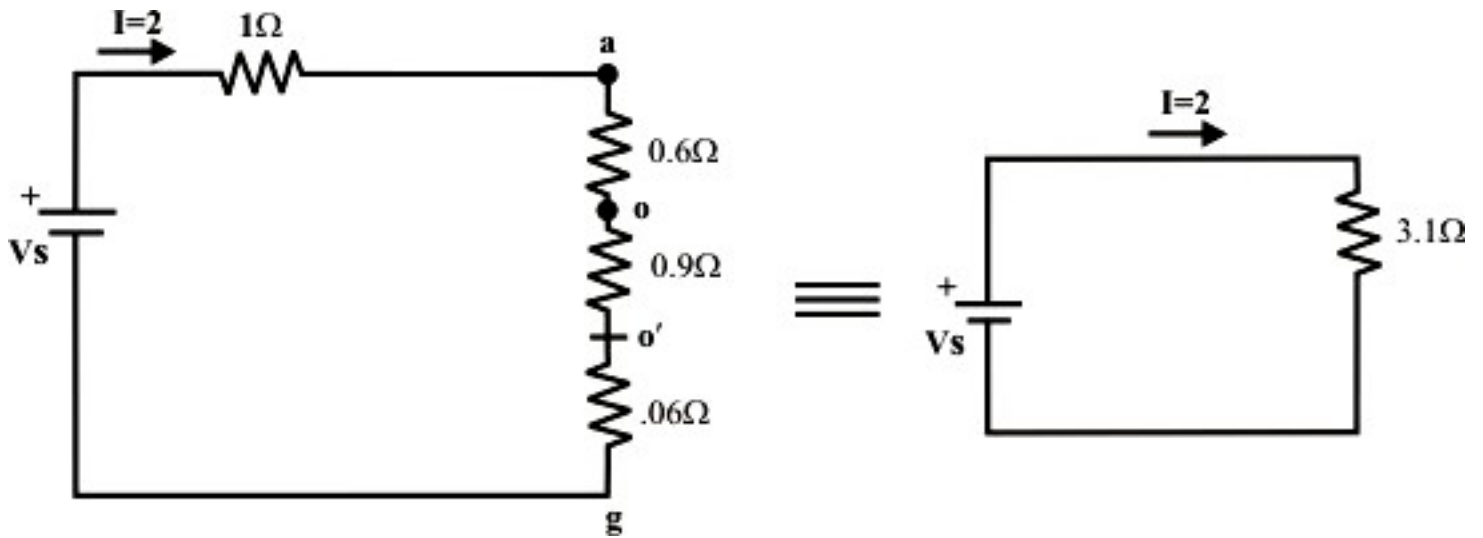
$$R_{ao} = \frac{3 \times 1}{5} = 0.6 \Omega; R_{co} = \frac{1 \times 1}{5} = 0.2 \Omega; R_{do} = \frac{3 \times 1}{5} = 0.6 \Omega$$

Similarly, for the  $\Delta$ -connected network 'e-f-g' the equivalent the resistances of Y-connected network are calculated as

$$R_{eo'} = \frac{3 \times 1}{5} = 0.6 \Omega; R_{go'} = \frac{3 \times 1}{5} = 0.6 \Omega; R_{fo'} = \frac{1 \times 1}{5} = 0.2 \Omega$$

Now the original circuit is redrawn after transformation and it is further simplified by applying series-parallel combination formula.





The source  $V_s$  that delivers  $2A$  current through the circuit can be obtained as  $V_s = I \times 3.2 = 2 \times 3.1 = 6.2 \text{ Volts}$ .

**Example:** L.6.2 Determine the equivalent resistance between the terminals  $A$  and  $B$  of network shown in fig.6.4 (a).

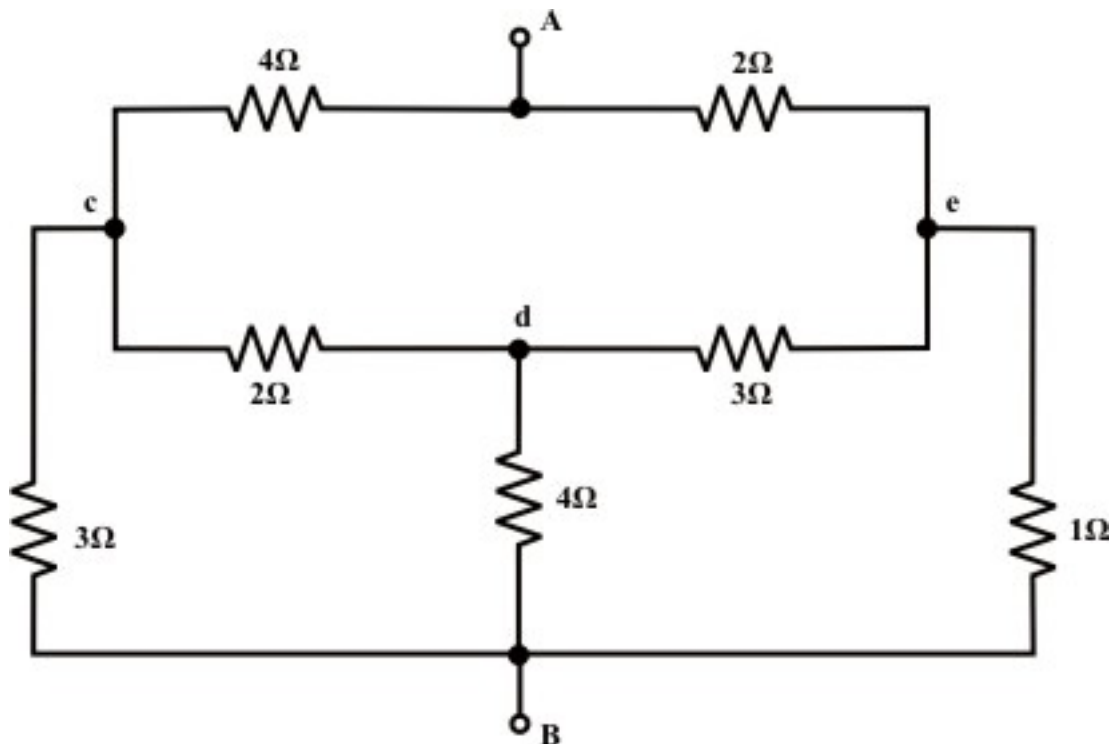


Fig. 6.4 (a)

**Solution:**

A 'Δ' is substituted for the 'Y' between points c, d, and e as shown in fig.6.4(b); then unknown resistances value for Y to Δ transformation are computed below.

$$R_{cB} = 2 + 4 + \frac{2 \times 4}{3} = 8.66\Omega; R_{eB} = 3 + 4 + \frac{4 \times 3}{2} = 13\Omega; R_{ce} = 2 + 3 + \frac{2 \times 3}{4} = 6.5\Omega$$

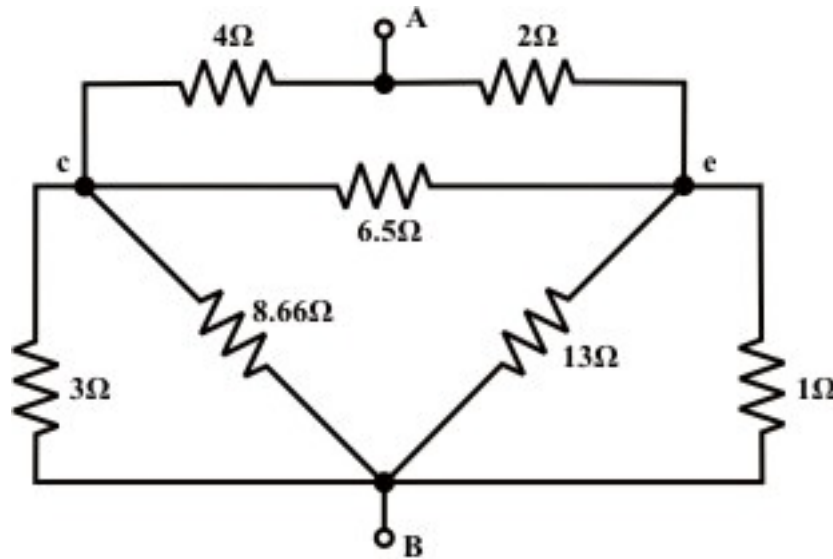


Fig. 6.4 (b)

Next we transform 'Δ' connected 3-terminal resistor to an equivalent 'Y' connected network between points 'A'; 'c' and 'e' (see fig.6.4(b)) and the corresponding Y connected resistances value are obtained using the following expression. Simplified circuit after conversion is shown in fig. 6.4(c).

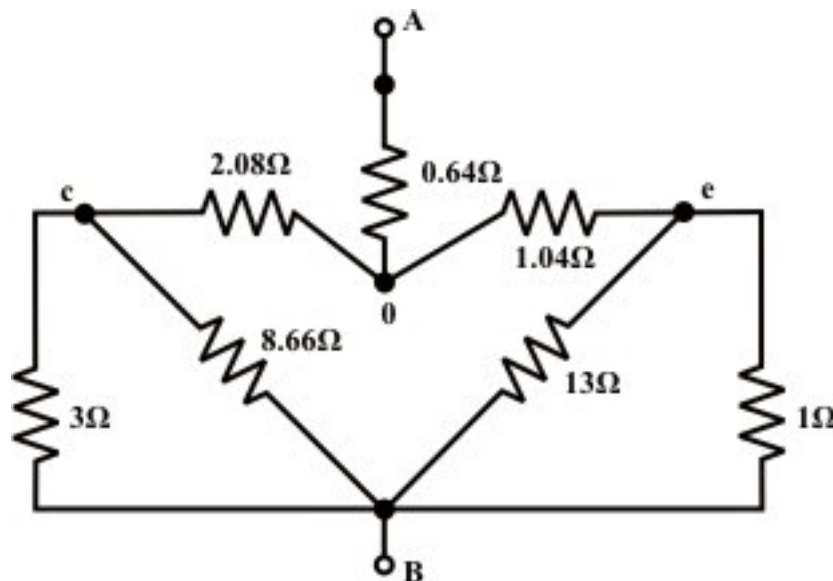


Fig. 6.4 (c)

$$R_{Ao} = \frac{4 \times 2}{4 + 2 + 6.5} = 0.64\Omega; \quad R_{co} = \frac{4 \times 6.5}{4 + 2 + 6.5} = 2.08\Omega; \quad R_{eo} = \frac{6.5 \times 2}{4 + 2 + 6.5} = 1.04\Omega;$$

The circuit shown in fig.6.5(c) can further be reduced by considering two pairs of parallel branches  $3 \parallel 8.66$  and  $13 \parallel 1$  and the corresponding simplified circuit is shown in fig.6.4(d).

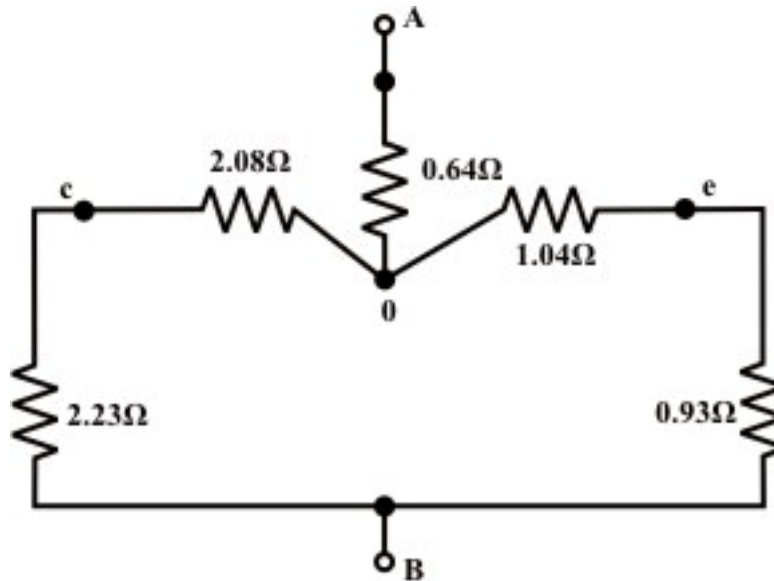
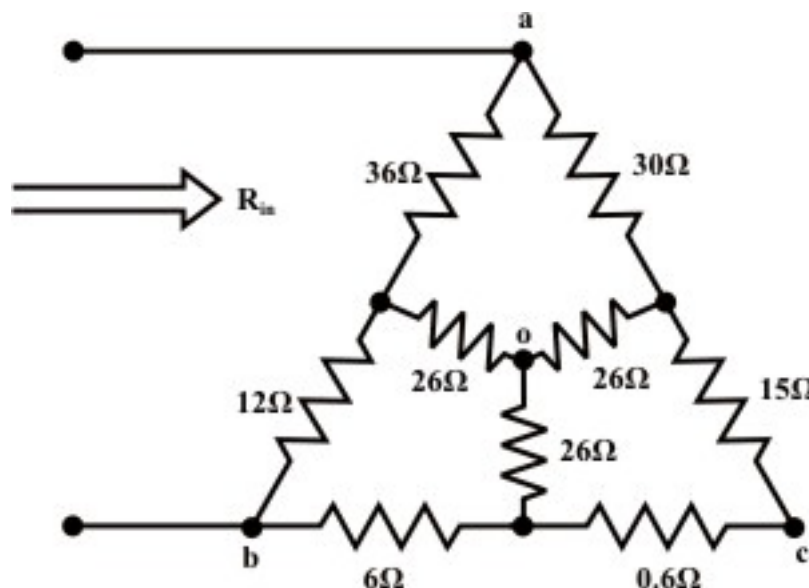


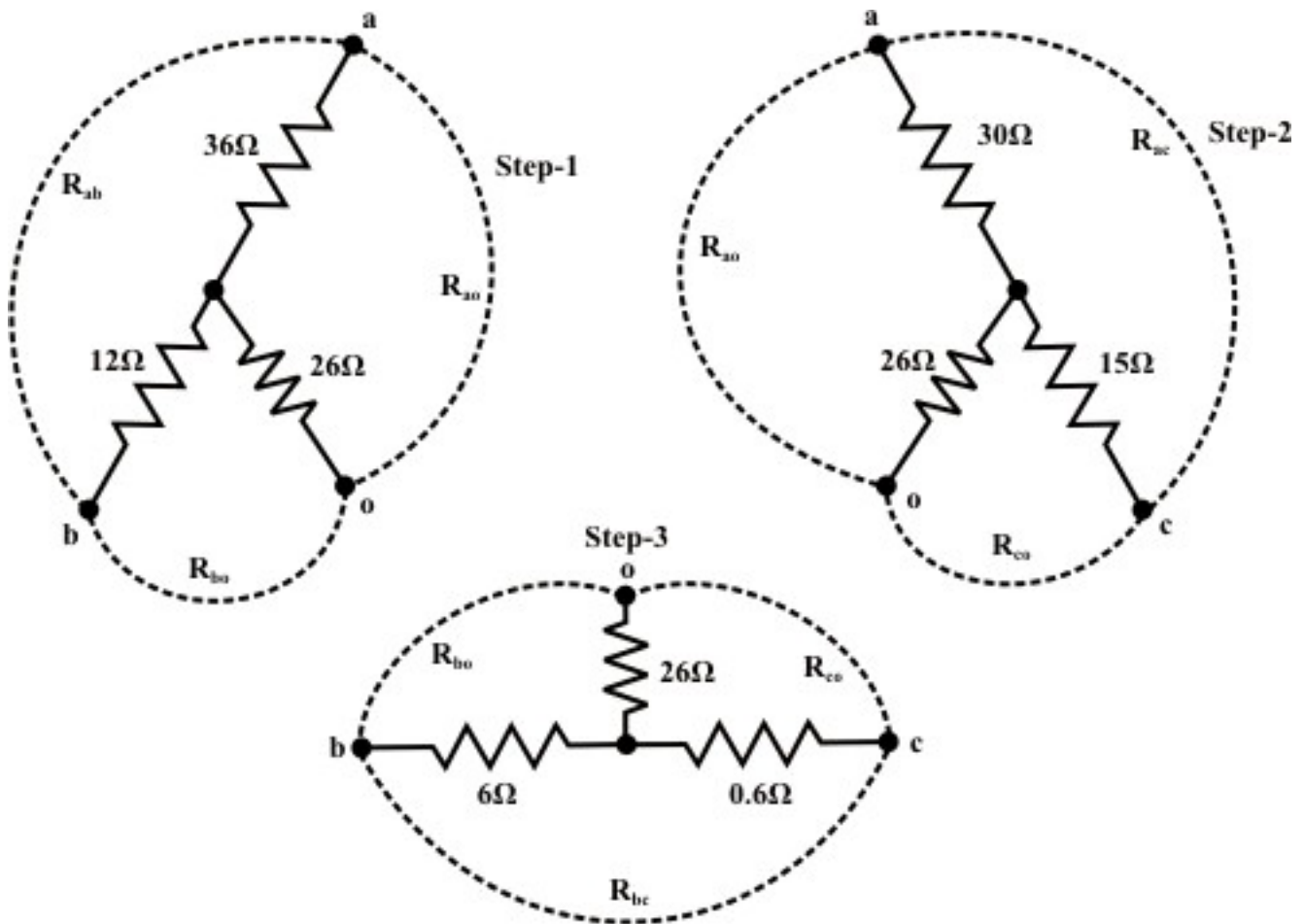
Fig. 6.4 (d)

Now one can find the equivalent resistance between the terminals 'A' and 'B' as  $R_{AB} = (2.23 + 2.08) \parallel (1.04 + 0.93) + 0.64 = 2.21\Omega$ .

**Example:** L.6.3 Find the value of the input resistance  $R_{in}$  of the circuit.



**Solution:**



Y connected network formed with the terminals a-b-o is transformed into  $\Delta$  connected one and its resistance values are given below.

$$R_{ab} = 36 + 12 + \frac{36 \times 12}{26} = 64.61\Omega ; \quad R_{bo} = 12 + 26 + \frac{26 \times 12}{36} = 46.66\Omega$$

$$R_{ao} = 26 + 36 + \frac{26 \times 36}{12} = 140\Omega$$

Similarly, Y connected networks formed with the terminals 'b-c-o' and 'c-a-o' are transformed to  $\Delta$  connected networks.

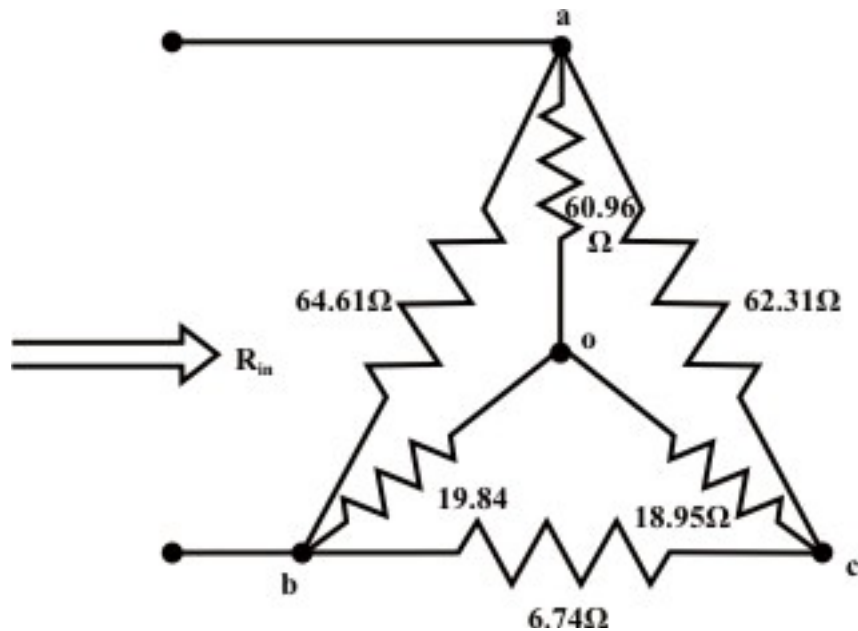
$$R_{bc} = 6 + 0.6 + \frac{6 \times 0.6}{26} = 6.738\Omega ; \quad R_{co} = 0.6 + 26 + \frac{0.6 \times 26}{6} = 29.2\Omega$$

$$R_{bo} = 6 + 26 + \frac{6 \times 26}{0.6} = 34.60\Omega$$

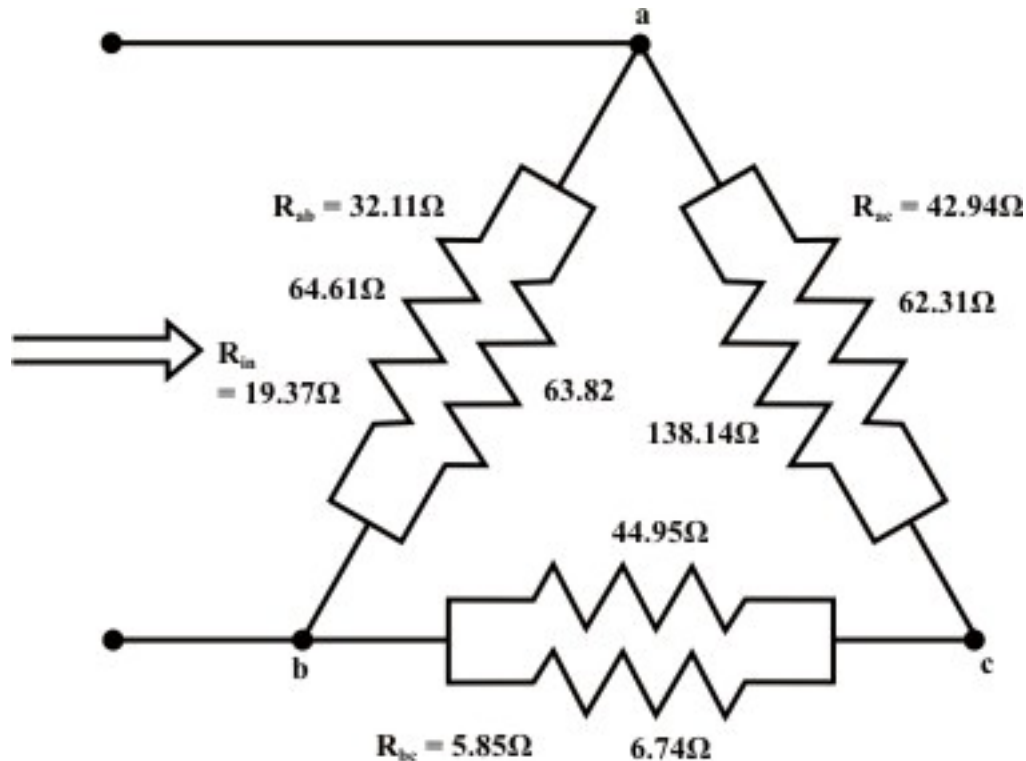
$$\text{and, } R_{co} = 15 + 26 + \frac{15 \times 26}{30} = 54.00\Omega ; \quad R_{ao} = 30 + 26 + \frac{30 \times 26}{15} = 108\Omega$$

$$R_{ac} = 30 + 15 + \frac{30 \times 15}{26} = 62.31 \Omega$$

Note that the two resistances are connected in parallel ( $140 \parallel 108$ ) between the points 'a' and 'o'. Similarly, between the points 'b' and 'o' two resistances are connected in parallel ( $46.66 \parallel 34.6$ ) and resistances  $54.0 \Omega$  and  $29.2 \Omega$  are connected in parallel between the points 'c' and 'o'.



Now  $Y$  connected network formed with the terminal 'a-b-c' is converted to equivalent  $\Delta$  connected network.



Now, 
$$R_{in} = \frac{(R_{ac} + R_{bc})R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = 19.37\Omega$$

**Remarks:**

- If the  $\Delta$  or  $Y$  connected network consists of inductances (assumed no mutual coupling forms between the inductors) then the same formula can be used for  $Y$  to  $\Delta$  or  $\Delta$  to  $Y$  conversion (see in detail 3-phase ac circuit analysis in Lesson-19).
- On the other hand, the  $\Delta$  or  $Y$  connected network consists of capacitances can be converted to an equivalent  $Y$  or  $\Delta$  network provided the capacitance value is replaced by its reciprocal in the conversion formula (see in detail 3-phase ac circuit analysis in Lesson-19).

**Example:** L.6.4 Find the equivalent inductance  $R_{eq}$  of the network (see fig.6.5(a)) at the terminals 'a' & 'b' using  $Y-\Delta$  &  $\Delta-Y$  transformations.



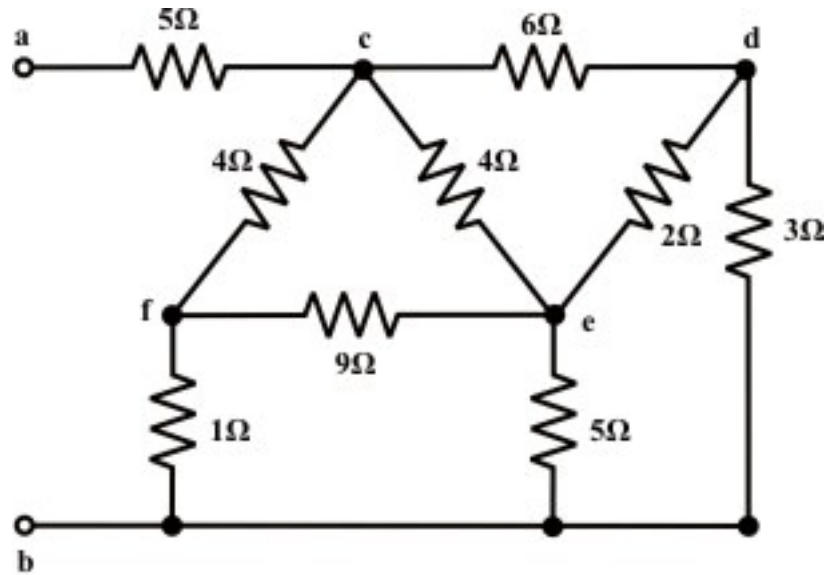


Fig. 6.5(a)

**Solution:** Convert the three terminals (c-d-e)  $\Delta$  network (see fig.6.5(a)) comprising with the resistors to an equivalent  $Y$ -connected network using the following  $\Delta-Y$  conversion formula.

$$R_{co} = \frac{6 \times 4}{12} = 2\Omega; R_{do} = \frac{6 \times 2}{12} = 1\Omega; \text{ and } R_{eo} = \frac{2 \times 4}{12} = 0.666\Omega$$

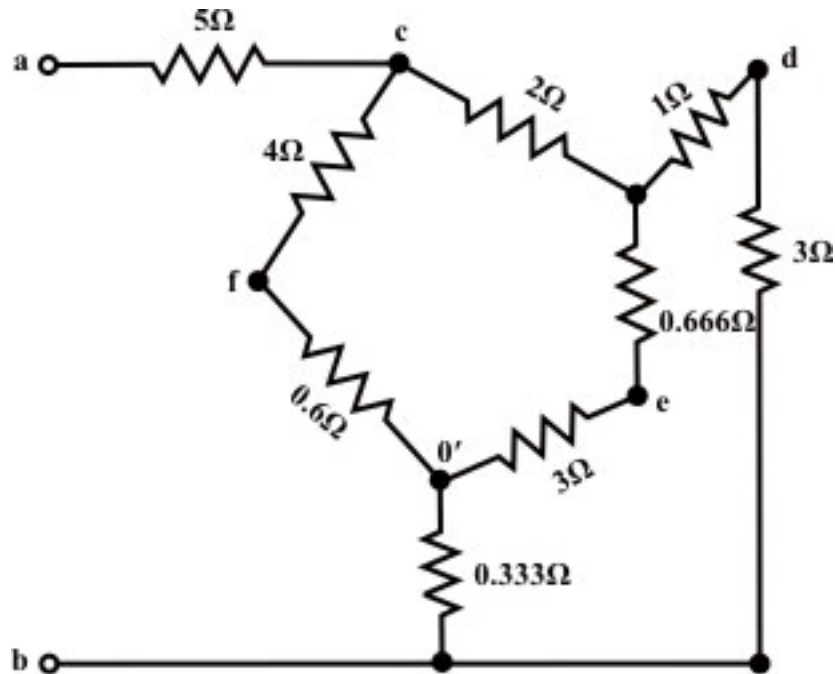
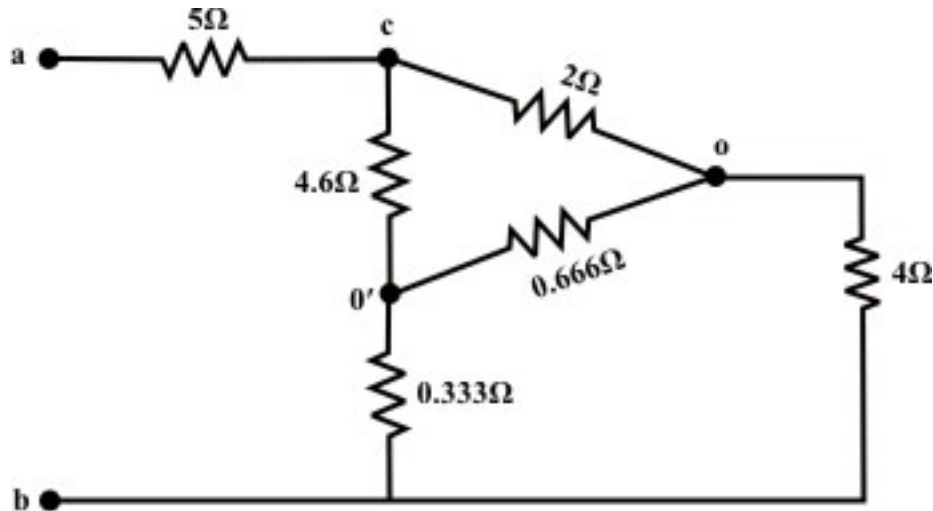


Fig. 6.5(b)



**Fig. 6.5(c)**

Similarly, the  $\Delta$ -connected network ( f-e-b) is converted to an equivalent  $Y$ -connected Network.

$$R_{fo'} = \frac{1 \times 9}{15} = 0.6\Omega; R_{eo'} = \frac{5 \times 9}{15} = 3\Omega; \text{ and } R_{bo'} = \frac{1 \times 5}{15} = 0.333\Omega$$

After the  $\Delta - Y$  conversions, the circuit is redrawn and shown in fig.6.5(b). Next the series-parallel combinations of resistances reduces the network configuration in more simplified form and it is shown in fig.6.5(c). This circuit (see fig.6.5(c)) can further be simplified by transforming  $Y$  connected network comprising with the three resistors ( $2\Omega$ ,  $4\Omega$ , and  $3.666\Omega$ ) to a  $\Delta$ -connected network and the corresponding network parameters are given below:

$$R_{co'} = 2 + 3.666 + \frac{2 \times 3.666}{4} = 7.5\Omega; R_{cb} = 2 + 4 + \frac{2 \times 4}{3.666} = 8.18\Omega;$$

$$\text{and } R_{bo'} = 4 + 3.666 + \frac{4 \times 3.666}{2} = 15\Omega$$

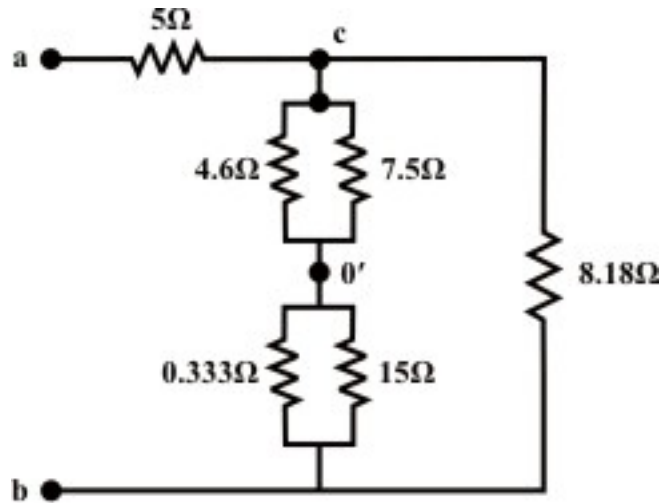


Fig. 6.5(d)

Simplified form of the circuit is drawn and shown in fig.6.5(d) and one can easily find out the equivalent resistance  $R_{eq}$  between the terminals 'a' and 'b' using the series-parallel formula. From fig.6.5(d), one can write the expression for the total equivalent resistance  $R_{eq}$  at the terminals 'a' and 'b' as

$$\begin{aligned} R_{eq} &= 5 + [(4.6 \parallel 7.5) + (0.333 \parallel 15)] \parallel 8.18 \\ &= 5 + [2.85 + 0.272] \parallel 8.18 = 5 + (3.122 \parallel 8.18) \\ &= 7.26\Omega \end{aligned}$$

### L.6.3 Test Your Understanding

[Marks: 40]

T.1 Apply  $Y-\Delta$  or  $\Delta-Y$  transformations only to find the value of the Current  $I$  that drives the circuit as shown in fig.6.6. [8]

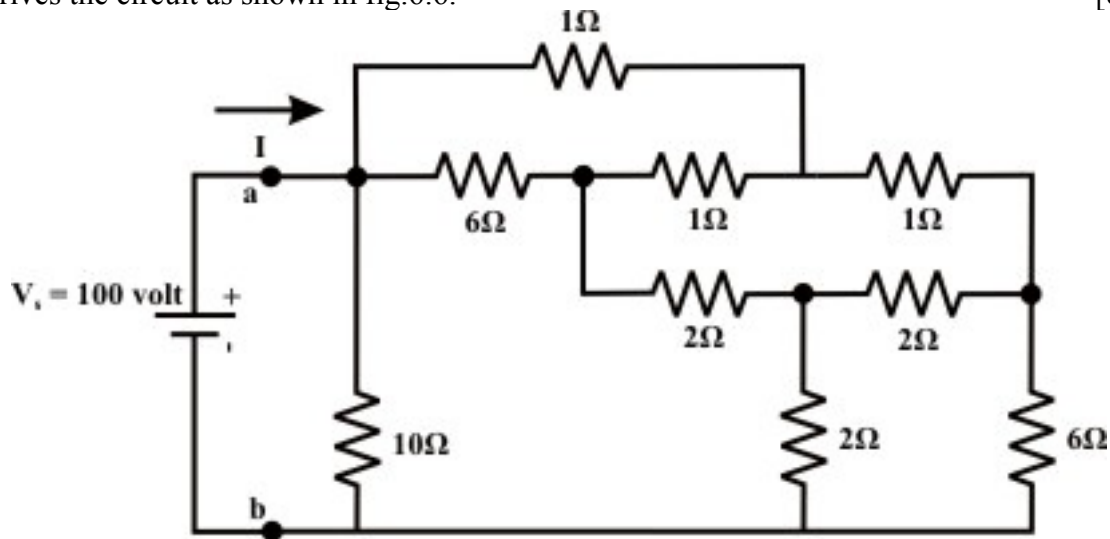


Fig. 6.6

(ans:  $10.13\Omega$ )

T.2 Find the current  $I$  through  $4\Omega$  resistor using  $Y-\Delta$  or  $\Delta-Y$  transformation technique only for the circuit shown in fig.6.7. [10]

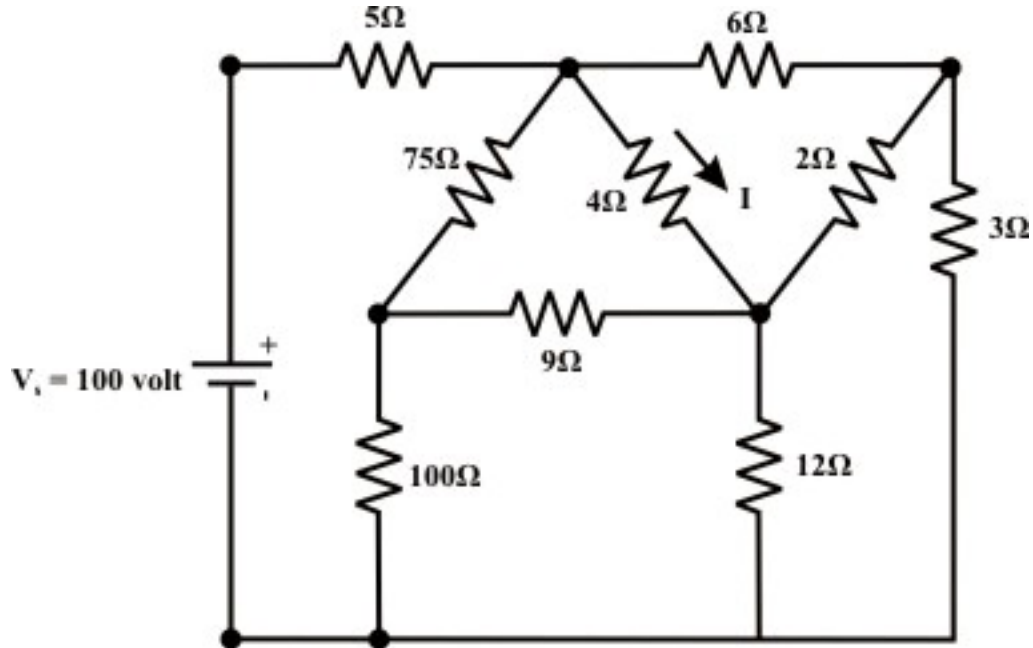


Fig. 6.7

(ans:  $7.06\text{ A}$  )

T.3 For the circuit shown in fig.6.8, find  $R_{eq}$  without performing any conversion. [4]

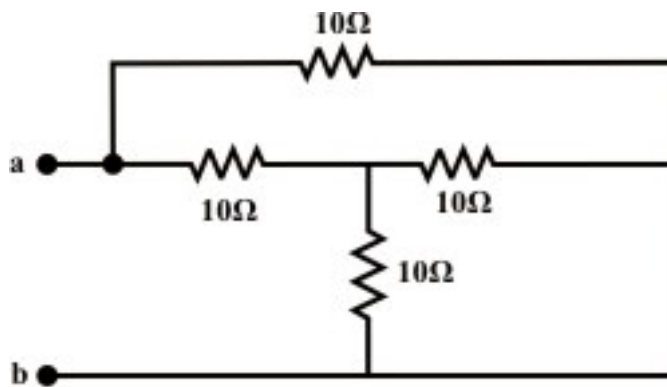
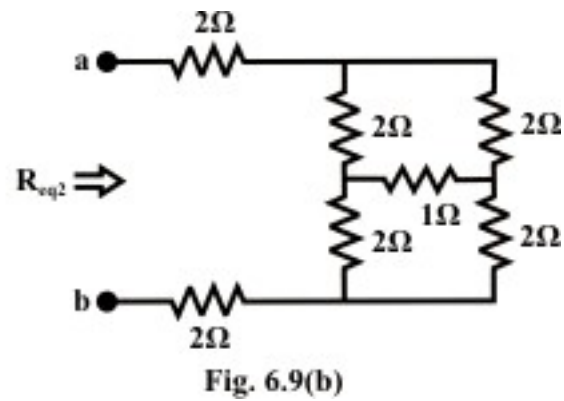
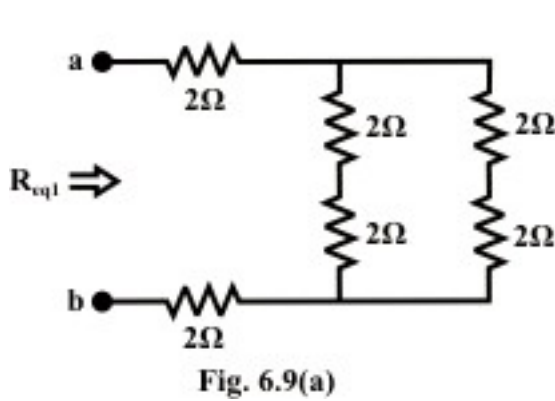


Fig. 6.8

(Ans.  $6\Omega$ )

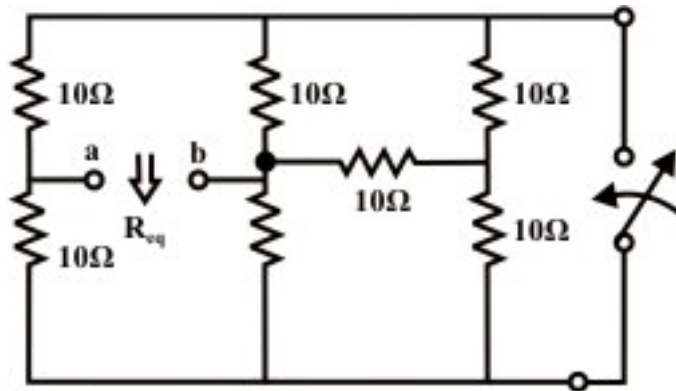
T.4 For the circuit shown in fig.6.9, calculate the equivalent inductance  $R_{eq}$  for each circuit and justify your answer conceptually. [6]



(ans.  $R_{eq1} = R_{eq2}$ )

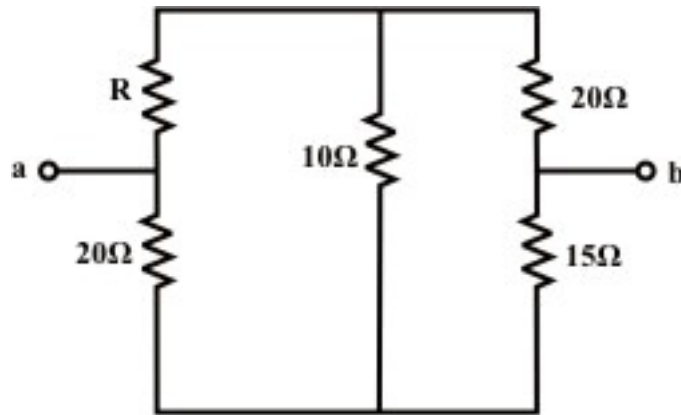
T.5 Find the value of  $R_{eq}$  for the circuit of fig.6.10 when the switch is open and when the switch is closed. [4]

(Ans.  $R_{eq} = 8.75\Omega$  ;  $R_{eq} = 7.5\Omega$ )



T.6 For the circuit shown in fig.6.11, find the value of the resistance ' $R$ ' so that the equivalent capacitance between the terminals 'a' and b' is  $20.57\Omega$ . [6]

(Ans.  $30\Omega$ )



**Fig. 6.11**

T.7  $Y-\Delta$  or  $\Delta-Y$  conversion is often useful in reducing the ----- of a resistor network ----- to the beginning nodal or mesh analysis. [1]

T.8 Is it possible to find the current through a branch or to find a voltage across the branch using  $Y-\Delta/\Delta-Y$  conversions only? If so, justify your answer. [1]

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